SYNCHROTRON SIDEBANDS OF AN RF-INDUCED RESONANCE*

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Polarized proton beams encounter many depolarizing resonances during acceleration to high energies. Many of these resonances are also found to have synchrotron sideband resonances, caused mainly by the energy modulated spin tune[1]. In addition to the spin tune modulation, strong synchrotron sidebands around RF-induced resonances could be caused by the energy modulation of the RF perturbation itself. This paper discusses how the two energy modulation effects could cause interference of the synchrotron sideband resonances.

In a ring without Siberian snakes, the proton's spin precesses around the vertical field of the ring's dipoles. The spin precession rate is given by the spin tune ν_s, which is proportional to the Lorentz energy factor γ, ($\nu_s = G\gamma$, where $G = 1.793$ is proton's gyromagnetic anomaly). An RF magnetic field can induce a spin depolarizing resonance with the strength given by a Fourier integral,

$$\varepsilon = \frac{1}{2\pi}\int_0^{2\pi} \phi_{RF} \cdot e^{iG\gamma\theta} d\theta \ , \tag{1}$$

where $\phi_{RF} = 2\pi\varepsilon_0 \cos\left(\frac{f_{RF}}{f_{circ}}\theta\right)$ is the angle of spin perturbation by the RF field. Resonant depolarization occur when RF frequency match the spin precession frequency $f_{RF} = f_{circ}(n \pm \nu_s)$. The synchrotron energy oscillations modulate the spin tune with the synchrotron frequency ν_{syn}; as a consequence additional harmonics appear in the spectrum of the spin precession frequencies. In the presence of the spin tune modulation, integrated phase of the spin precession can be written

$$\psi = \int_0^{\theta} (G\gamma_0 + G\Delta\gamma\cos(\nu_{syn}\theta))d\theta = G\gamma_0\theta + \frac{G\Delta\gamma}{\nu_{syn}}\sin(\nu_{syn}\theta) \ , \tag{2}$$

where $\Delta\gamma = \beta^2\gamma(\Delta p/p)$ is the amplitude of the energy oscillations. The frequency spectrum of the spin precession can be expanded[2]

$$e^{i\psi} = e^{iG\gamma\theta} e^{i\frac{G\Delta\gamma}{\nu_{syn}}\sin(\nu_{syn}\theta)} = \sum_{m=-\infty}^{\infty} J_m\left(\frac{G\Delta\gamma}{\nu_{syn}}\right) \cdot e^{i(G\gamma + m\nu_{syn})\theta} \ , \tag{3}$$

where $J_m(x)$ is the m^{th} Bessel function of the first kind. Thus, along with

* Supported by grants from the U.S. Department of Energy and National Science Foundation.

the main RF-induced resonance the spin tune modulation creates synchrotron sidebands at $k = G\gamma \pm \frac{f_{RF}}{f_{circ}} + m\nu_{syn}$. The strength of such sideband resonances is given by,

$$\varepsilon_m = \frac{\varepsilon_0}{2} J_m \left(\frac{G\Delta\gamma}{\nu_{syn}} \right) \ . \tag{4}$$

Besides energy dependent spin tune, the RF-induced resonance equation, $k = G\gamma \pm \frac{f_{RF}}{f_{circ}}$, includes beam circulation frequency, which is also modulated by the synchrotron energy oscillations,

$$\frac{\Delta f_{circ}}{f_{circ}} = -\eta \frac{\Delta p}{p} \cos(\nu_{syn}\theta) \ , \tag{5}$$

where $\eta = \alpha - \gamma^{-2}$ depends on the beam energy and on the ring's orbit compaction factor $\alpha = \frac{p}{L}\frac{dL}{dp}$. The energy dependent circulation frequency makes the RF-induced spin perturbation modulated by the synchrotron energy oscillations, which can also cause synchrotron sidebands. Expressing the energy spread $\Delta\gamma$ in terms of $\Delta p/p$, the synchrotron sideband strength of an RF-induced resonance could be written,

$$\varepsilon_m = \frac{\varepsilon_0}{2} J_m \left(\frac{\Delta p}{p\nu_{syn}} (G\beta^2\gamma \pm \eta \frac{f_{RF}}{f_{circ}}) \right) \ . \tag{6}$$

The two energy modulation effects combined result in stronger synchrotron sidebands around $G\gamma + k$ RF-induced resonance while the sidebands around $k - G\gamma$ resonance should be weaker.

Such an interference effect of the synchrotron sidebands was recently observed at IUCF [3]. For a 104.1 MeV polarized proton beam, the synchrotron sidebands of the $G\gamma - 1$ resonance were found to be much stronger than the sidebands of the $3 - G\gamma$ resonance. The observed strengths of the main resonances were equal. The data is shown in Fig. 1. Using Eq. (6) one can estimate that, synchrotron sidebands of the $G\gamma - 1$ resonance are expected to be about 3 times stronger than the sidebands of the $3 - G\gamma$ resonance. The data suggests that the synchrotron sidebands have different depth while their widths are similar. This could be an indication of a large synchrotron tune spread; then, the weaker synchrotron sidebands would be depolarizing only a fraction of the beam while the stronger sidebands would depolarize the whole beam.

A similar interference effect exists for intrinsic resonances ($k = G\gamma \pm \nu_z$), where the energy dependence of the betatron tune ν_z is determined by the ring's chromaticity function. Then, one could correct the synchrotron sidebands of a strong intrinsic resonance by adjusting the ring's chromaticity function. This method was successfully used in the KEK PS [4]; however, its application is limited to low energies by the size of the chromaticity function.

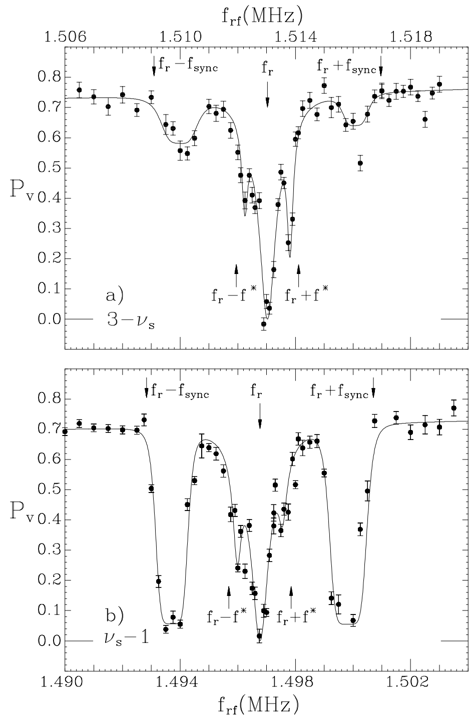

Figure 1: The vertical proton polarization measured at 104.1 MeV [2] is plotted against the rf solenoid frequency. Fig. 1a shows $3 - G\gamma$ resonance and Fig. 1b shows the $G\gamma - 1$ resonance. The curves are Lorenzian fits to the data. Note the very narrow resonances on the sides of the primary resonance; these are due to remaining first harmonics injection rf structure in the beam with frequency f^*.

References

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