BOUNDS ON SPIN 1/2 - SPIN 1/2 HELICITY AMPLITUDES

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A study of bounds on helicity amplitudes provides important information related to the evaluation of polarization and the understanding of spin-dependent reactions. In *pp* scattering the asymmetry maximum in the low momentum transfer region is sensitive to the imaginary helicity single-flip amplitude which we bound using the method of Lagrange multipliers.

Introduction

Recent indications suggest that the component quarks of a proton contribute the surprisingly small fraction of $\Delta\Sigma = 0.29 \pm 0.06$ to its spin. Other sources of the spin of a proton additional to total quark spin contribution include the quark and gluon angular momenta L_q, L_g and Δg which represents the contribution from the spin of gluonic fields. QCD provides estimates for such additional contributions but, to test such predictions experimentally, a proton beam whose polarization is known to within 5% is required in high energy, high transverse momentum, quark-gluon Compton scattering, dijet and charmonium processes. One method of proton polarimetry uses the single spin asymmetry at small collision angles.

Asymmetry and Partial Wave Amplitudes

Assuming that hadronic double flip amplitudes ϕ_2, ϕ_4 and $\phi_1 - \phi_3$ are negligible, we may write the asymmetry [1]

$$\frac{m\,A_N}{\sqrt{-t}} = \frac{(\kappa_p - 2\,\mathrm{Im}\,r_5)\,\frac{t_c}{t} + 2\,\mathrm{Re}\,r_5 - 2\rho\,\mathrm{Im}\,r_5}{\left(\frac{t_c}{t}\right)^2 - 2(\rho+\delta)\frac{t_c}{t} + 1 + \rho^2}\,, \tag{1}$$

where the average helicity [2] amplitude is $\phi_+(s,t) = (\phi_1 + \phi_3)/2$. Amplitudes are conventionally written with hadronic and electromagnetic elements $\phi_j + e^{i\delta}\phi_j^e$, $(j = 1, \ldots, 5)$, involving a Coulomb phase shift δ. Introduce the ratio $r_5(s,t)$ scaled by a kinematic factor of $m/\sqrt{-t}$,

$$r_5(s,t) = \frac{\phi_5(s,t)}{\mathrm{Im}\,\phi_+(s,t)}\frac{m}{\sqrt{-t}}\,. \tag{2}$$

Interference between $\phi_+^e(s,t)$ and $\phi_+(s,t)$ is most prominent in the Coulomb Nuclear Interference (CNI) region when $t = t_c$ where $t_c = -8\pi\alpha/\sigma_{tot} \approx 0.0012\,(\mathrm{GeV}/c)^2$. Here $\kappa_p + 1 = \mu = 2.793$ is the proton's magnetic moment, m is its mass and $\rho = \mathrm{Re}\,\phi_+/\,\mathrm{Im}\,\phi_+$. Note that in the CNI region the asymmetry is sensitive to $\mathrm{Im}\,r_5$. For the scattering of two equal mass spin 1/2 particles at CM energy $\sqrt{s}$ and momentum $k = \sqrt{s-4m^2}/2$, the five independent helicity [2,3,4] amplitudes $\phi_1,\ldots,\phi_5$ can be written as a linear combination of the s-channel partial wave amplitudes $f_i^J = r_i^J + i a_i^J$ $(i = 0,1,11,22,21)$ and as a function of $d^J_{\lambda\mu}(z)$ where $z = \cos\theta = 1 + t/2k^2$. The total and elastic cross sections can be written as

$$A_0 = \frac{k^2}{\pi}\sigma_{tot} = \sum_J (2J+1)\left(a_0^J + a_1^J + a_{11}^J + a_{22}^J\right)\,, \tag{3}$$

$$\Sigma_{el} = \frac{k^2}{\pi}\sigma_{el} = \sum_J (2J+1)\left\{\left|f_0^J\right|^2 + \left|f_1^J\right|^2 + \left|f_{11}^J\right|^2 + \left|f_{22}^J\right|^2 + 2\left|f_{21}^J\right|^2\right\}\,. \tag{4}$$

We write the slope of the imaginary helicity non-flip amplitude at $t=0$ as $B = 16k^3 s^{-1/2} d/dt\,\mathrm{Im}\,\phi_+(s,t)\big|_{t=0}$,

$$B = \sum_J (2J+1)\left\{J\,(J+1)\left(a_0^J + a_1^J + a_{11}^J + a_{22}^J\right) - \left(a_1^J + a_{22}^J\right)\right\} \tag{5}$$

where

$$\frac{4\pi}{k\sqrt{s}\,\sigma_{tot}}\,\frac{d}{dt}\,\mathrm{Im}\,\phi_+(s,t)\bigg|_{t=0} = g(s) = \frac{d}{dt}\ln\mathrm{Im}\,\phi_+(s,t)\bigg|_{t=0}\,. \tag{6}$$

The imaginary single-flip amplitude, expressed with Jacobi polynomials [5],

$$\mathrm{Im}\,\phi_5(s,t) = -\frac{\sqrt{s}}{4k}\sum_J (2J+1)\sqrt{\frac{J+1}{J}}\,(1-z^2)^{1/2} a_{21}^J P_{J-1}^{(1,1)}(z)\,, \tag{7}$$

indicates for high energies where a large J approximation is appropriate, that

$$\frac{\mathrm{Im}\,\phi_5(s,t)}{\sin\theta}\bigg|_{\theta=0} = -\frac{\sqrt{s}}{4k}\sum_J (2J+1)\sqrt{J(J+1)}\,a_{21}^J \approx -\sum_J J^2 a_{21}^J \tag{8}$$

which for $\theta \sim 0$ reads

$$\mathrm{Im}\,r_5 = \frac{4m\pi}{k^2\sqrt{s}\,\sigma_{tot}}\,\frac{\mathrm{Im}\,\phi_5}{\sin\theta}\,. \tag{9}$$

The partial wave amplitudes satisfy the unitarity inequality [4],

$$u^J = a_0^J - (a_0^J)^2 - (r_0^J)^2 \geq 0\,. \tag{10}$$

Helicity single-flip amplitude bound

We can find an upper bound on $|\mathrm{Im}\, r_5|_{t=0}$ using unitarity, the total cross section, the elastic cross section and the slope of the imaginary non-flip amplitude at $t=0$ as constraints. Introduce the auxiliary function [6],

$$\begin{aligned}
\mathcal{L} = \left.\frac{\mathrm{Im}\,\phi_5}{\sin\theta}\right|_0 &+ \alpha\left[\Sigma_{el} - 2\sum_J J\left\{\left|f_0^J\right|^2 + \left|f_1^J\right|^2 + \left|f_{11}^J\right|^2 + \left|f_{22}^J\right|^2 + 2\left|f_{21}^J\right|^2\right\}\right] \\
&+\beta\left[B - 2\sum_J J^3\left(a_0^J + a_1^J + a_{11}^J + a_{22}^J\right)\right] + 2\sum_J J\lambda_J u^J \\
&-\gamma\left[A_0 - 2\sum_J J\left(a_0^J + a_1^J + a_{11}^J + a_{22}^J\right)\right] \qquad (11)
\end{aligned}$$

and optimize $\mathrm{Im}\,\phi_5/\sin\theta|_{\theta=0}$ by differentiating $\mathcal{L}$ with respect to the real and imaginary partial wave amplitudes. Noting Eq. (10) we choose the unitarity class $I \equiv \{J|u^J > 0, \lambda_J = 0\} : 0 < a_0^J < 1$ to obtain $r_i^J = 0$ for all i, $a_0^J = a_1^J = a_{11}^J = a_{22}^J = -(\beta J^2 + \gamma)/2\alpha$, $a_{21}^J = -J/8\alpha$, where α, β, $\gamma > 0$ and $0 < a_0^J < 1$ implies $J^2 < \gamma/\beta$. Re-constructing A_0, Σ_{el}, B and $\mathrm{Im}\,\phi_5/\sin\theta|_{\theta=0}$ from Eqs. (3), (4), (5) and (8), using Eq. (9) we find $|\mathrm{Im}\, r_5|_{t=0} \leq m\sqrt{g}\sqrt{36\pi g\sigma_{el}/\sigma_{tot}^2 - 1}$. For proton proton scattering, assuming $g \sim \sigma_{tot}^2/32\pi\sigma_{el}$, we have the bound $|\mathrm{Im}\, r_5|_{t=0} \leq 0.85$, $\sqrt{s} = 50\,\mathrm{GeV}$. As 0.85 is less than $\kappa_p/2 = 0.896$, we conclude that the asymmetry A_N in the CNI region is non-zero and positive for like-sign fermion fermion collisions.

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