Photon polarization in radiative electroweak process, $e^-e^+ \to \nu\overline{\nu}\gamma$

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It is shown that the transition amplitude of radiative electroweak process $e^-e^+ \to \nu\overline{\nu}\gamma$ can be reduced and factorized into a simpler form in the Standard Model. It is used to investigate the photon polarization in $e^-e^+ \to \nu\overline{\nu}\gamma$.

In our previous work [1], it has been shown that, in the Standard Model (SM), the $\gamma e^- \to W^-\nu_e$ process can be treated mathematically as a special case of the $\gamma e^- \to Z^0 e^-$ process. This method is applied to the $\nu_e e^- \to \nu_e e^-\gamma$ process which is described by five Feynman diagrams [2,3] and three W-mediated diagrams can be reduced into the two Z^0-mediated diagrams. Explicitly the transition amplitude of the process $e^-(p_1)e^+(p_2) \to \nu_e(k_1)\overline{\nu_e}(k_2)\gamma(k)$ can be obtained as follows [4],

$$\begin{aligned}
\mathcal{M}_{e^-e^+\to\nu_e\overline{\nu}_e\gamma} &= \frac{eg_W^2}{8\cos^2\theta_W}\frac{1}{(2k_1\cdot k_2 - m_Z^2)} \\
&\times\overline{v}(p_2)\Bigg[\frac{\gamma^\mu(\not\epsilon_\gamma^*\not k + 2p_1\cdot\epsilon_\gamma^*)}{2k\cdot p_1}\left[E_{1L}(1-\gamma_5)+\epsilon_R(1+\gamma_5)\right] \\
&\quad+\frac{(\not\epsilon_\gamma^*\not k - 2p_2\cdot\epsilon_\gamma^*)\gamma^\mu}{2k\cdot p_2}\left[E_{2L}(1-\gamma_5)+\epsilon_R(1+\gamma_5)\right] \\
&\quad+2\gamma^\mu(k_2-p_1)\cdot\epsilon_\gamma^* A_L(1-\gamma_5)\Bigg]u(p_1)\overline{u}(k_1)\gamma_\mu(1-\gamma_5)v(k_2),
\end{aligned} \tag{1}$$

where

$$E_{1L} = \epsilon_L + A_L[2p_1\cdot(k+k_1)+m_W^2], \tag{1a}$$

$$E_{2L} = \epsilon_L + A_L[2p_2\cdot(k+k_2)+m_W^2], \tag{1b}$$

$$A_L = -\cos^2\theta_W\frac{(2k_1\cdot k_2 - m_Z^2)}{(2p_1\cdot k_1+m_W^2)(2p_2\cdot k_2+m_W^2)}. \tag{1c}$$

The third term in the square bracket of Eq. (1) is necessary for the transition amplitude to have gauge invariance after the $\epsilon_L(1-\gamma_5)$ term is modified. The process $e^-e^+ \to \nu\overline{\nu}\gamma$ can be described essentially as Fig. 1.

From Fig. 1 one can say that the photon energy distribution becomes large at $(2k_1 \cdot k_2 - M_Z^2) = 0$. This comes from the Z^0-mediated diagrams out of five original Feynman diagrams. Away from these Z^0 peaks, the contribution from other diagrams becomes large as the incident energy increases. The transition amplitude $\mathcal{M}_{\gamma e \to \nu_e \overline{\nu}_e e}$ for the $\gamma(k)e^-(p_1) \to \nu_e(k_1)\overline{\nu}_e(k_2)e^-(p_2)$ process can be obtained from Eq. (1) for $\mathcal{M}_{e^-e^+ \to \nu_e \overline{\nu}_e \gamma}$ by replacing $k, p_2, v(p_2)$, and ϵ_γ^* by $-k, -p_2, u(p_2)$, and ϵ_γ, respectively. Also the transition amplitude $\mathcal{M}_{\nu_e e \to \nu_e e \gamma}$ for the $\nu_e(k_2)e(p_1) \to \nu_e(k_1)e(p_2)\gamma(k)$ can be obtained similarly from Eq. (1) of $\mathcal{M}_{e^-e^+ \to \nu_e \overline{\nu}_e \gamma}$ by replacing $k_2, p_2, v(k_2)$, and $v(p_2)$ by $-k_2, -p_2, u(k_2)$, and $u(p_2)$, respectively.

Finally the photon polarization in the process $e^-e^+ \to \nu\overline{\nu}\gamma$ can be described by the Stokes' parameters of the outgoing photon. Using values of photon polarization vector $\epsilon^\mu(\lambda) = \frac{1}{\sqrt{2}}(n_1^\mu + i\lambda n_2^\mu)$ as given by Landau-Lifschitz series [5], the explicit values of Stokes parameters $\xi_i (i = 1, 2, 3)$ can be obtained. Even though analytic calculation might be possible, we present here numerical results of Stokes parameters at $\sqrt{s} = 60, 92, 100, 150, 200$, and 500 GeV, which are obtained by means of GRASE program [6].

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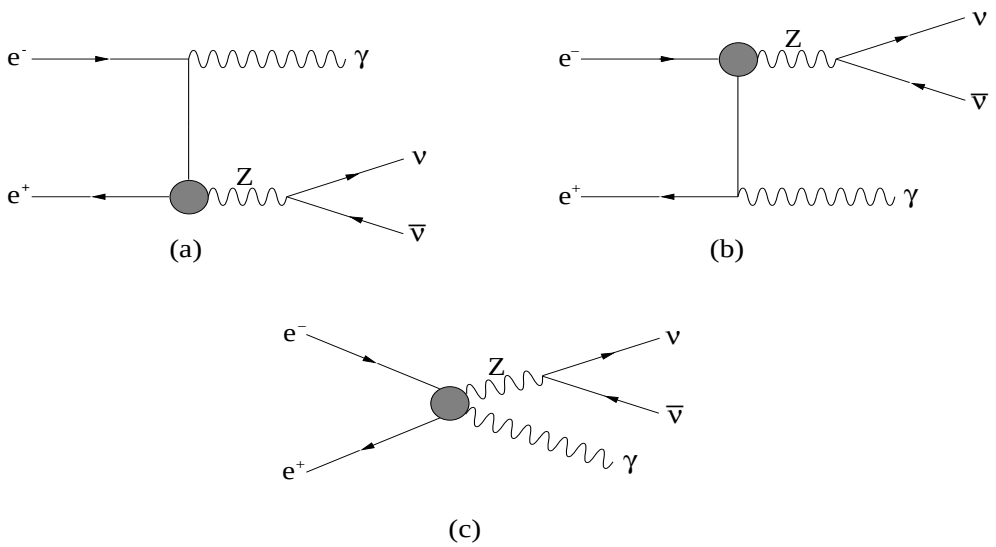

Figure 1: Feynman diagrams for $e^- e^+ \to \nu\overline{\nu}\gamma$.

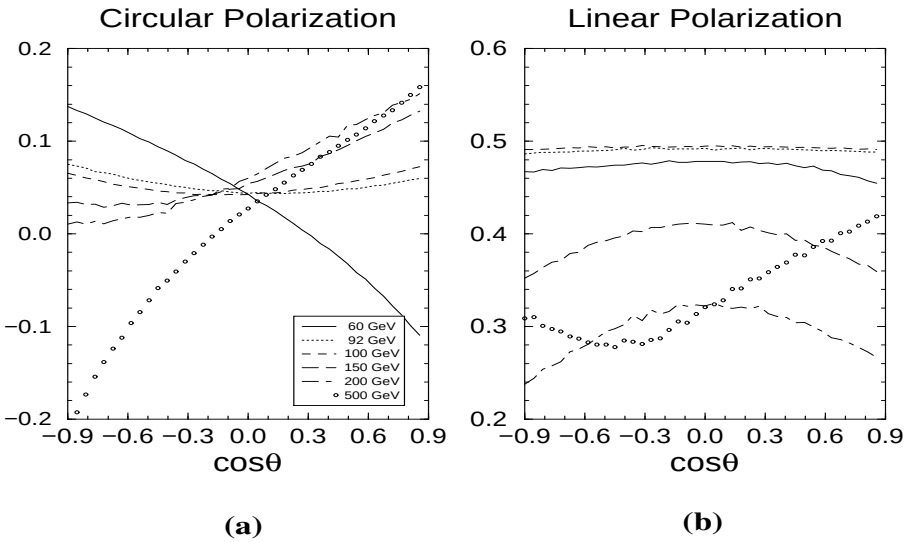

Figure 2: Stokes' parameters for circular polarization(ξ_2) and linear polarization(ξ_2) of photon at various incident electron energies.