DEPOLARIZATION OF THE PROTON BEAM DUE TO CORRECTING ELEMENTS IN THE NUCLOTRON RING AT JINR

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1 Introduction

The Nuclotron ring at JINR is a unique accelerator with a superconducting magnet lattice, and the investigation of the polarization characteristics of the accelerated beam is one of very important tasks in experimental physics. During the proton beam acceleration (as well as the others particles) problems appear, which are connected with spin resonances. The traditional way of elimination of the depolarization is a compensation of the spin resonance strengths. But the problem of the spin motion correction correlates with the problem of the orbit motion correction. Here a question appears: on the one hand, is it possible to use standard correcting elements for this purpose and what could be their optimal location along the orbit? On the other hand, how should we use the orbit motion correction not to destroy the spin motion?

Let Q denote a strength of correcting element which disturbs the vertical motion of the particle:

$$y'' + g_y y = Q\,.$$

Then the strength w_k of the spin resonance $\nu \equiv \gamma\,\frac{q_a}{q_0} = \nu_k$ will be

$$w_k = \frac{1}{2\pi R}\int\limits_0^{2\pi} Q F_\nu \exp\left(i\,\nu_k\theta\right)\,d\theta$$

where

$$F_\nu = \frac{\exp\left(-i\,\nu_k\,\theta\right)}{2}\cdot\left[f_y^*\int\limits_{-\infty}^{\theta}\left(\left(\nu_k^2-1\right)f_y'\,K_y - i\,\nu_k\,\tfrac{q}{q_0}\,f_y\,K_y^2\right)\exp\left(i\,\nu_k\,\widetilde{K_y}\right)\,d\theta - \right.$$
$$\left. - f_y\int\limits_{-\infty}^{\theta}\left(\left(\nu_k^2-1\right)f_y'^{\,*}\,K_y - i\,\nu_k\,\tfrac{q}{q_0}\,f_y^*\,K_y^2\right)\exp\left(i\,\nu_k\,\widetilde{K_y}\right)\,d\theta\right]$$

is a function of the spin response[1] which is defined by the ideal structure of the Nuclotron ring[2] with radius R and with superperiodicity $p = 8$; f_x and f_y are radial and vertical Floquet functions; $\widetilde{K_y} = \int\limits_0^\theta K_y \, d\theta$.

The function F_ν ($F_\nu(\theta + \frac{2\pi}{p} n) = F_\nu(\theta)$) is a "response" of the system to the perturbation in a form of a Dirac's δ-function, that is if $Q = 2\pi \, R \, \delta(\theta - \theta_c)$, then the resonance strength is proportional to the response function:

$$w_k = F_\nu(\theta_c) \exp(i \, \nu_k \, \theta_c) \, .$$

Thus, the effect of the correcting elements depends very much on their values, interlocation and the response function^a.

2 Spin resonances driven by correcting dipoles

In the case of one correcting dipole all integer spin resonances in the energy range from 20 MeV to 11 GeV become possible:

$$\nu = k \, , \qquad 2 \le k \le 22 \, .$$

As an example, let us consider the resonance at the injection energy. Figure 1 shows the response function for $k = 2$. The other resonances are analyzed by the same way.

The effect of the correcting dipole on the spin is minimal if it is located in the points of the minimum of the response function and is maximal in the points of the maximum of $\mid F_\nu \mid$.

For the correcting dipole 0.3 m long with a field of $H_x = 0.13$ T $Q = R \, \frac{H_x}{\langle H_y \rangle}$, and we have the resonance strength $\mid w_k \mid = 1.4 \cdot 10^{-4}$ for $k = 2$ ($E = 0.11$ GeV).

So it follows that at the injection energy the degree of depolarization (in the case if the magnetic field is increased with the speed of 1 T/sec) will be less than 2%. In the case of the crossing of spin resonance at the maximum energy the beam will be practically depolarized. The maximum vertical distortion of the orbit at the maximum energy of the Nuclotron ring will be about 0.25 cm.

But if two correcting dipoles are located in the points with azimuths $\theta = \theta_1$ and $\theta = \theta_2$ according the conditions:

$$k(\theta_1 - \theta_2) + \arg F_\nu(\theta_1) - \arg F_\nu(\theta_2) = n\pi \, , \qquad \mid H_x^1 \, F_\nu(\theta_1) \mid = \mid H_x^2 \, F_\nu(\theta_2) \mid$$

^{a}Note, that in this paper we do not take into account the depolarization near the spin intrinsic resonances: $\nu = pn \pm \nu_y$.

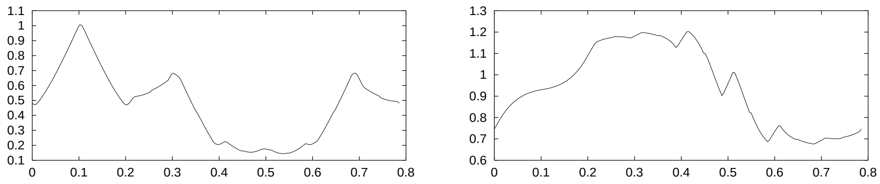

Figure 1: Module (left) and argument (right) of the response function for resonance $\nu = 2$

then two dipoles compensate completely each other with the elimination of the depolarization for resonances $\nu = k$ (if the correspondent choice of the field signs is made).

3 Conclusions

By the same way we can analyze the cases of correcting quadrupoles and sextupoles.

In the case if two correcting quadrupoles are located in the points where $\theta = \theta_1$ and $\theta = \theta_2$ according the conditions:

$$k(\theta_1 - \theta_2) + \arg \frac{F_\nu(\theta_1)\, f_x^p(\theta_1)}{F_\nu(\theta_2)\, f_x^p(\theta_2)} = n\pi\,,$$

$$\left|\frac{\partial H_x}{\partial x}\right|_1 \mid F_\nu(\theta_1) \mid \cdot \mid f_x(\theta_1) \mid = \left|\frac{\partial H_x}{\partial x}\right|_2 \mid F_\nu(\theta_2) \mid \cdot \mid f_x(\theta_2) \mid\,,$$

two quadrupoles compensate completely each other and there is no any depolarization during the resonance crossing $\nu = k + \nu_x$. Here $f_x^p = f_x e^{-i\nu_x\theta}$ is the periodic Floquet function. We have to replase f_x^p on f_x^{p*} for resonances $\nu = k - \nu_x$ in the last formulas.

In conclusion we notice, that the response function may help to analyze the most difficult situation, when it is necessary to conserve the polarization of the beam in accelerators.

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References

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