PROTON POLARIMETRY BY UNDULATOR RADIATION

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We show how spin light from an undulator can be used to measure the spin polarization of a proton beam in a high energy circular collider. We propose to perform left-right asymmetry measurements, employing lock-in amplification techniques.

1 Introduction

A contribution to synchrotron radiation due to the magnetic moment of the particle, or "spin light", was originally suggested by Ternov [1], then studied by several Authors [2,3] and finally measured in polarized electrons [4]. We will show how using undulator radiation, proton spin polarimetry is possible [5].

Electromagnetic spin polarimetry is complementary to polarimetry based on nuclear effects. It measures the *magnetic moment* of the particle. Then, the spin vector, defined in the particle rest frame, can be calculated by a relativistic transformation.

In an undulator, the electric dipole radiation due to a moving electric charge in an external field and the magnetic dipole radiation due to a moving magnetic moment have opposite polarization and parity angular distribution, so left-right asymmetry measurement can be used. In a superconducting transverse undulator, the radiation spectrum falls in the infra-red for energies of hundreds of GeV, and in the soft X-rays for multi-Tev. It can be measured as right-left asymmetry by detectors sensitive to the polarization of the radiation. Lock-in amplification techniques and possibly an optical resonant cavity can enhance the signal-to-noise ratio.

2 Undulator Radiation.

In a semi-classical treatment [6,8], the radiation power at **P** emitted by a moving particle at **Q** is proportional to the modulus-squared of the vector potential $\vec{A}$

$$\frac{dP(t)}{d\Omega} = \left|\vec{A}(t)\right|^2 = \left|\vec{A}(\omega)\right|^2 \tag{1}$$

For the electric- and magnetic- radiation ($\vec{\mu}$, magnetic moment) it is

$$\begin{cases} \vec{A}^{\varepsilon}(t) = U\vec{a}^{\varepsilon}(t), & \vec{a}^{\varepsilon} = \hat{n}\times[\hat{n}\times\vec{\beta}] \\ \vec{A}^{\mu}(t) = \eta U\vec{a}^{\mu}(t), & \vec{a}^{\mu}(t) = \hat{n}\times\left[\hat{s}+\hat{n}\times[\vec{\beta}\times\hat{s}]\right] \end{cases} \tag{2}$$

U is a coefficient dependent on the Units. In the PRF, g is the spin anomaly

$$\vec{\mu} = g\frac{e\hbar}{4m_o c}\hat{s}, \quad \eta = g\frac{\hbar\omega}{4m_o c^2} \tag{3}$$

Since η is small, only an interference term in Eq.(1) survives and it is

$$|\vec{A}(\omega)|^2 \simeq \frac{U^2}{2\pi}\left[a^\varepsilon(\omega)^2 + 2\eta a^\varepsilon(\omega)a^\mu(\omega)\right] \tag{4}$$

Consider a transverse undulator of period λ_0 and *horizontal* field B_0. ($\hat{x}$ is radial, $\hat{y}$ vertical). Let the angles of the unit vector $\hat{n}$ be θ, azimuth, and ϕ, latitude. The wavelength at harmonic n and the photon flux at $n = 1$ in a bandwidth $\Delta\omega/\omega$ from the undulator are [7,8].

$$\lambda_n = \lambda_0\frac{1+\frac{1}{2}K^2+\gamma^2(\theta^2+\phi^2)}{2n\gamma^2}, \quad K = \frac{e}{2\pi mc}B_0\lambda_0, \quad n_{ph} = \pi N\alpha\frac{I}{e}\frac{\Delta\omega}{\omega}K^2 \tag{5}$$

N, no. of undulator periods, I, beam current, α, fine structure constant.

To second order in the angles and K/γ, the radiation components, polarized along $\hat{p}$ ($\perp$ to $\hat{y}$ and to $\hat{n} = \mathbf{P} - \mathbf{Q}$), and along $\hat{\sigma}$ ($\perp$ to $\hat{n}$ and $\hat{p}$), respectively, are

$$\begin{cases} a_p^\varepsilon(\omega) = -\frac{1}{\sqrt{2\pi}}\hat{p}\cdot\int e^{i\psi}\,\vec{\beta}\,dt, \quad a_\sigma^\varepsilon(\omega) = -\frac{1}{\sqrt{2\pi}}\hat{\sigma}\cdot\int e^{i\psi}\,\vec{\beta}\,dt \\ a_p^\mu(\omega) = \frac{1}{\sqrt{2\pi}}\left(\hat{\sigma}\cdot\int e^{i\psi}\,\hat{s}\,dt - \hat{p}\cdot\int e^{i\psi}\,[\vec{\beta}\times\hat{s}]\,dt\right) \\ a_\sigma^\mu(\omega) = -\frac{1}{\sqrt{2\pi}}\left(\hat{p}\cdot\int e^{i\psi}\,\hat{s}\,dt + \hat{\sigma}\cdot\int e^{i\psi}\,[\vec{\beta}\times\hat{s}]\,dt\right) \end{cases} \tag{6}$$

In Eq.(6), use the precession angle of the spin (with $\omega_0 = 2\pi c/\lambda_0$)

$$\mu = (1+G\gamma)\frac{K}{\gamma}\sin\omega_0 t, \quad G = g-2 = 3.58 \ \ \text{(protons)} \tag{7}$$

Note that only integrals of even functions survive, showing peaks, $\delta(n)$, at frequencies n given by Eq.(5). Only consider harmonic n =1, and obtain

$$\begin{cases} A_p^\varepsilon = -2R\gamma^2\phi\theta, \quad A_\sigma^\varepsilon = -R(1-2\gamma^2\phi^2) \\ A_p^\mu = \eta R\left\{\theta(1+2\gamma^2\phi^2)s_x - \phi\left[G\gamma - \gamma^2(\theta^2-\phi^2)\right]s_y \right. \\ \qquad\qquad \left. - \left[(\theta^2-\phi^2)G\gamma - 2\gamma^2\phi^2\right]s_z\right\} \\ A_\sigma^\mu = -\eta R\left\{\phi\left[1-\gamma^2(\theta^2-\phi^2)\right]s_x - \theta(G\gamma - 2\gamma^2\phi^2)s_y + 2\gamma^2\theta\phi s_z\right\} \end{cases} \tag{8}$$

$$R = iU\,\frac{K}{\gamma}\,f(1), \quad f(n) = \sqrt{\frac{\pi}{2}}\,\frac{N}{\omega_0}\,\delta(n) \tag{9}$$

Table 1: Undulator and Collider Parameters.

λ_0	$\mathbf{B}_0$	N	K	$\hbar\omega_0$
0.20 m	3.2 T	20	0.0326	6.20 μeV
δ	τ_{obs}	ε^*	β_T	I
10^{-4}	10 min	10 μm	1 m	150 mA

3 A practical example for a proton collider

Consider a super-conducting undulator and a high energy proton machine (Table 1). Re-write the coefficient η as

$$\eta = \frac{\hbar n \gamma^2 \omega_0}{2mc^2} = \frac{\lambda_{eff}}{\lambda_0} n\gamma^2, \quad \lambda_{eff} = \frac{\pi\hbar}{mc} = 1.33 \times 10^{-15} \text{ m} \tag{10}$$

The beam produces a radiation pattern with angular spread (with ε^*, norm. emittance, β^*, Twiss beta function at the insertion)

$$\tilde{\phi}, \tilde{\theta} = \sqrt{\frac{\varepsilon^*}{\beta\gamma\,\beta^*}} \tag{11}$$

For n=1, in the horizontal ($\phi = 0$), the total radiation power is

$$|A_p|^2 \simeq 0, \quad |A_\sigma|^2 \simeq R^2(1 - 2\eta\theta G\gamma s_y) \tag{12}$$

In σ-, a left-right measurement is possible, since the interference term, proportional to the vertical s_y is an odd function of θ. As detailed calculations show, this term peaks at $\sim \tilde{\theta}$ [Eq.(11)], then

$$\Re = \frac{r-l}{r+l} \simeq \left(\frac{4}{\pi}\right) \frac{\lambda_{eff}}{\lambda_0} G\,\tilde{\theta}\,\gamma^3 s_y \tag{13}$$

The signal-to noise ratio S/N could be increased using a lock-in amplifier, since the collider beam is in a series of bunches, each containing any chosen pattern of polarization. The signal increases as n_b, total number of bunches, the noise increase as $\sqrt{n_b}$, then S/N increases as $\sqrt{n_b}$.

Now, throw in the main limiting factor to S/N in a right-left measurement, i.e. the error of the center beam by an angle $\delta\tilde{\theta}$, and finally write (results in Table 2)

$$S/N = \left(\frac{4}{\pi}\right) \frac{\sqrt{n_b}}{\delta} \Re \tag{14}$$

Table 2: $n = 1,\ \phi = 0$.

γ (ring)	200 (RHIC)	800 (HERA)	7000 (LHC)
λ [nm]	2500	156	2.04
$n_b/turn$	110	174	2800
S/N	0.152	2.93	1220
n_{ph} [sec^{-1}]	2.28×10^{13}	=	=

At $n = 2$, it is the p-component apt to measure s_y. However, this harmonic is much less intense than $n = 1$, making $n = 2$ a less attractive choice.

Number of photons emitted in the central cone of $n = 1$ [Eq. (5)], if all colliders carry the same beam current, is also given in Table 2.

4 Conclusions

At high energy we can measure the proton magnetic moment with a superconducting undulator. Signal-to-noise ratio rapidly increases with energy. Since an asymmetry measurement is needed, possibly the most critical parameter is the relative error in the central angle of the beam with respect to the optical center of the detector assembly. An optical feedback system to reduce this error may be needed.

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