# POLARIZATION OF HIGH-ENERGY SPIN-1/2 PARTICLES IN MAGNETIC FIELD

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The investigation of the high-energy spin-1/2 particle interaction with a magnetic field is efficient means for studying properties of particles and verifying the quantum electrodynamics investigation results. To describe the interaction, the Foldy-Wouthuysen (FW) representation could be successfully used. Its use is very convenient in describing the particle polarization, and the methods of the transformation to this representation for particles in a magnetic field are well known [1,2]. One of the FW features is an even (quasidiagonal) form of relativistic wave equations and all operators. This makes the separation procedure for even parts of the operators unnecessary and allows to confine oneself to a single spinor. In this representation the relations between the operators are similar to the relations between the respective classical values. This simplifies the quasiclassical transition considerably.

In some cases the interaction of spin-1/2 particles with uniform magnetic field can be described exactly [1–4]. At the rotation of the Dirac particle, not possessing an anomalous magnetic moment (AMM), in an uniform magnetic field, its polarization vector rotates at the same angular velocity, and its projections onto the directions of magnetic field and particle momentum remain constant [5]. We can show that the polarization vector projections onto the three axes of the cylindrical coordinate system remain unchanged at the particle motion.

The interaction of particles having AMM with electromagnetic fields is describes by the Dirac-Pauli equation [6]. Inspite of the relative AMM smallness (for electrons, positrons, and muons), this moment considerably affects the particle polarization. The FW representation use allows to find the exact equation of the spin motion of the particles, having AMM, in an uniform magnetic field and to describe exactly the motion of particles and the spin rotation relative to momentum [4]. The energy levels of particles with AMM are also known exactly [3]. If the particle moves in the plane, perpendicular to the field strength $\mathbf{H}$, the equation of the spin motion has the form [4]:

$$\frac{d\mathbf{\Pi}}{dt}=\frac{1}{2}\left([\mathbf{\Omega\Pi}]-[\mathbf{\Pi\Omega}]\right), \quad \mathbf{\Omega}=\mathbf{\Omega}_D-2\beta\mu'\mathbf{H}, \quad \mathbf{\Omega}_D=-\beta(E_+-E_-)\frac{\mathbf{H}}{H},$$
$$E_\pm=\sqrt{\boldsymbol{\pi}^2+m^2\pm eH}, \quad \boldsymbol{\pi}=\mathbf{p}-e\mathbf{A},$$

where $\mathbf{\Pi} = \beta\mathbf{\Sigma}$ is the three-dimensional polarization operator in the FW representation, $\beta$ and $\mathbf{\Sigma}$ are the Dirac matrices, $\mathbf{p} = -i\nabla$ is the momentum operator, $\mathbf{\Omega}$ and $\mathbf{\Omega}_D$ are the operators of the angular velocity of spin precession for particles having AMM and Dirac particles.[a] The exact equation for the operator of the angular velocity of the particle rotation has the form:

$$\mathbf{o} = \mathbf{\Omega}_D.$$

The spin motion of a particle relative its momentum is characterized by the difference in angular velocities, $\mathbf{\Omega} - \mathbf{o}$. The nonlinear dependence of this value on $\mathbf{H}$ would be possible only when the initial equation is more general than the Dirac-Pauli equation.

The energy of the interaction of a particle with a magnetic field is, as a rule, considerably smaller than the total energy, and the motion of relativistic particles can be described quasiclassically. It has been noted in [7,8] that the direction, the spin projection onto which has a fixed value, is not parallel to the field strength vector at the motion of particles along a helix in a magnetic field. As is shown in the work [9], the direction, the spin projection onto which has a fixed (quantized) value, is not parallel to the magnetic field direction and has a constant orientation with respect to the axes of the cylindrical coordinate system related to the moving particle. This direction lies in the plane of the vectors $\mathbf{e}_\phi$ and $\mathbf{e}_z$ and makes the angle $\theta$, defined by the expression:

$$\sin\theta = \frac{(\gamma - 1)\sin 2\delta}{\sqrt{2\left[\gamma^2 + 1 - (\gamma^2 - 1)\cos 2\delta\right]}},$$

with the $z$ axis. Here $\gamma$ is the Lorentz factor, $\delta$ is the angle formed by the particle momentum vector and the $z$ axis. When $\gamma \gg 1$ $\theta \approx \pi/2 - \delta$. The difference of $\theta$ from zero could be found experimentally.

The interesting effect which could have the practical application arises for the particles moving in the plane perpendicular to an uniform magnetic field in the presence of axially symmetrical transverse magnetic field which is even comparatively weak. We assume that the transverse magnetic field is constant and directed along the $\mathbf{e}_\phi$ vector of the cylindrical coordinate system, the center of which coincides with the center of the circular trajectory of the particle. Though the resulting magnetic field is not uniform, the spin quantization takes place in this case as well. The direction, the spin projection onto which is quantized, lies in the plane of the vectors $\mathbf{e}_\phi$ and $\mathbf{e}_z$ and makes the angle $\theta$, defined by the expression [10]:

$$\tan\theta = \frac{g}{|g-2|\gamma}\left|\frac{H_\phi}{H_z}\right|$$

[a] The relativistic system of units $\hbar = c = 1$ is used.

with the $z$ axis. The effect is extremally strong for electrons, positrons and muons. Since in this case $\frac{g}{g-2} \approx \frac{2\pi}{\alpha} \sim 10^3$, then even at $H_\phi \sim 10^{-3} \cdot H_z$ particles are polarized at a significant angle to the $z$ axis and at $H_\phi = 10^{-2} \cdot H_z$, $\gamma = 1.2$ for electrons $\theta = 82°$, i.e. the direction of their polarization is almost orthogonal to the direction of the uniform field. The great value of the effect makes better considerably the possibility of its experimental observation. It can be comparatively easy detected in the investigation of the particle spin rotation. The phenomenon might be of practical significance, as even a comparatively weak transverse magnetic field has an essential influence on the Sokolov-Ternov effect of the radiation polarization of particles in a magnetic field. In this case the partial polarization of the particle beam along the spin quantization direction takes place. This fact could be used for obtaining of electron and positron beams partially polarized at an arbitrary angle to the momentum direction.

The general case of particles in an axially symmetrical non-uniform magnetic field has been investigated.

The investigations show that by the inclusion of the additional axially symmetrical transverse electric field, the conditions could be created when the particles having AMM obtain a fixed helicity, i.e. the spin orientation parallel or antiparallel to momentum. For electrons and positrons this phenomenon could be observed beginning from non-relativistic velocities of particles to the energies of the 10 MeV order.

## References


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