MASS SPECTRUM AND SPIN STRUCTURE OF MESONS IN THE STRING QUARK MODEL

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The relativistic quantum string quark model proposed earlier, is applied to all mesons, from pion to Υ, lying on the leading Regge trajectories (i.e. to the lowest radial excitations in terms of the potential quark models). The model describes the meson mass spectrum, and comparison with measured meson masses allows one to determine the parameters of the model: current quark masses, universal string tension and phenomenological constants describing nonstring short-range interaction. The meson Regge trajectories are in general nonlinear; practically linear are only trajectories for light-quark mesons with nonzero lowest spins. The model predicts masses of many new high-spin mesons. In some cases the masses of new low-spin mesons are predicted by extrapolation of the phenomenological short-range parameters in the quark masses. In this way the model predicts the mass of $\eta_b(1S)(0^{-+})$ to be 9500 ± 30MeV, the mass of $B_c(0^-)$ to be 6400 ± 30MeV, and a new $K^*(1^-)$ meson is predicted with mass 1900 Mev. The relativistic wave functions of the composite mesons allow one to calculate the spin structure of polarized mesons. The average spin projections of the quarks within the ρ-meson are twice as small as the nonrelativistic quark model predictions. The spin structure of K^* reveals an 80% violation of the flavour $SU(3)$. These results may be relevant to understanding the "spin crises" for nucleons.

The naive quark model of hadron, attractive by its simplicity, is not so simple at a closer consideration. It is not relativistic since it contains a confinement potential proportional to the space distance. One can introduce a quasipotential dependent on distance and momentum which makes the wave equation Lorentz covariant, but then the phenomenological quasipotential is not simple. The model contains constituent quarks which are purely phenomenological notions. Their masses are not fundamental and can vary from mesons to hadrons and even from one meson to another. Their spins are not fundamental either and the "spin crises" for nucleons suggests they are different from 1/2 or the naive quark model is too naive.

In Ref.[1] an alternative string quark model (SQM) is proposed which does not contain separately either confinement potential or constituent quarks. Their physical origin, the gluon field, is taken into account explicitly in an approximation of the Nambu-Goto string. The string provides the confinement mechanism and, since the string is a physical object with its own energy-momentum and angular momentum, the quarks at the ends of the string are fundamental quarks with current masses and spin 1/2.

We consider here only the simplest string configuration — the rotating straight line, which is responsible for the leading Regge trajectories of mesons. The daughter trajectories correspond to vibrations of the string.

The model is quantized in accord and with Poincaré invariance and contains no tachyons.

The idea to use the straight-line string with massive quarks attached to its ends is not new (see References in [1]). What is new in the present approach is the consistent account of the quark spins and the canonical quantization.

The outline of the report is the following. We formulate the classical and quantum SQM; find out the wave functions of composite mesons and a spectral condition for their spins and masses; compare this condition with experimental meson masses, obtain the model parameters and make predictions of masses and other quantum numbers of new mesons; analyse spin structure of mesons; and finish by conclusions.

The classical Lagrangian of a meson in SQM consists of three parts

$$L = L_{str} + \Sigma L_i + L_{ss}, \tag{1}$$

the first one being the Nambu-Goto Lagrangian for a straight-line string

$$x(\tau,\sigma) = r(\tau) + f(\tau,\sigma)q(\tau), \tag{2}$$

which contains a string tension parameter a.

The second term in Eq. (1) is the sum of the Lagrangians for point-like spinning quarks[2] with current masses $m_i, i = 1,2$ having velocities of the string ends (which are perpendicular to the string direction). The quark Lagrangians contain quark spin variables $\xi_i^M (M = \mu, 5)$ and λ_i which anticommute with each other (and commute with other variables, including spin variables of the other quark).

The variables λ_i are Lagrange multipliers which yield spin constraints. The spin constraints must conserve. To ensure this conservation a third term L_{ss} must be added to the total Lagrangian (1) which restores its supersymmetry. This term contains no new parameters.

Besides these terms, a nonstring short-range interaction must enter into the meson Lagrangian. This interaction cannot be fully described at the classical level and will be taken into account in the quantum equations.

Introducing the total momentum p conjugate no r ($m = \sqrt{p^2}$ is the meson mass) and the internal momentum π conjugate to q and choosing gauge conditions $p\pi = 0$, $\pi^2 = -1$ we can rewrite the total Lagrangian (1) in the Hamilton form

$$L = -p\dot{z} - [\vec{k}\times\vec{L}]\dot{\vec{k}} - \frac{i}{2}\xi_{i(M)}\dot{\xi}_i^{(M)} -$$

$$- (\sqrt{\vec{J}^2} - K)C + \Sigma(p_i^{(\alpha)}\xi_{i(\alpha)} - m_i\xi_i^5)\tilde{\lambda}_i. \tag{3}$$

Here new (partly) independent variables have been introduced with the help of four orthonormal vectors e_α, $\alpha = 0, a$, $a = 1, 2, 3$

$$e_0 = p/m, \quad e_\alpha e_\beta = g_{\alpha\beta}, \tag{4}$$

so that

$$\pi = k^{(a)}e_a, \quad L = L^{(a)}e_a, \quad J = J^{(a)}e_a, \tag{5}$$

$$\xi_1 = \xi_i^{(\alpha)}e_\alpha, \quad \xi_i^{(5)} = \xi_i^5, \tag{6}$$

$$z^\mu = r^\mu + \frac{1}{2}\epsilon_{abc}e_a^\nu \frac{\partial e_{b\nu}}{\partial p_\mu} J^{(c)} + \frac{i}{m}\Sigma\xi_i^{(0)}\xi^{(a)}e_a^\mu, \tag{7}$$

where J and L are the total and orbital meson spins

$$J = L + S, \tag{8}$$

$$L_\mu = \epsilon_{\mu\nu\rho\sigma}p^\nu q^\rho \pi^\sigma/m, \quad S_\mu = \Sigma\epsilon_{\mu\nu\rho\sigma}p^\nu \xi_i^\rho \xi_i^\sigma/2im. \tag{9}$$

The functions K and $p_i^{(\alpha)}$ in Eq.(3) are given by

$$K = lm - G, \quad G = a\int_{-l_1}^{l_2} \sqrt{l^2 - y^2}dy + \Sigma m_i\sqrt{l^2 - l_i^2}\ , \tag{10}$$

$$l_i = \sqrt{l^2 + m_i^2/4a^2} - m_i/2a\ , \tag{11}$$

$$p_i^{(0)} = l\sqrt{am_i/l_i}; \quad \vec{p}_i = (-1)^i\sqrt{am_i l_i}\ \ \vec{k}. \tag{12}$$

where $\vec{k} = \{k^{(a)}\}$. In all these expressions l is a function of m given by the equation

$$\partial G/\partial l = m. \tag{13}$$

The first line on the r.h.s of Eq.(3) defines the Poisson brackets of our variables[3] and the second one yields Hamiltonian and three constraints between the variables. These constraints have zero Poisson brackets with each other what justifies the choice of L_{ss} in Eq.(1).

The quantization of our system is now straightforward. We replace the Poisson brackets by commutators or anticommutators for ξ's (which become γ-matrices[2]) and the constraint functions become operators annihilating the wave function. In the representation where p and $\vec{k}$ are diagonal the internal part of the wave function $\delta(p-p')\Psi_{\alpha\beta}(\vec{k})$ satisfies the equations (second particle is antiquark)

$$(\hat{p}_1 - m_1)\Psi = 0 \ , \quad \Psi(\hat{p}_2 + m_2) = 0 \ , \tag{14}$$

$$(\sqrt{\vec{J}^2} - K - \sum_{n=1}^{4} a_n P_n)\Psi = 0. \tag{15}$$

In the third equation a new term has been introduce to account for the short-range nonstring interaction. In this term a_n can depend on J, and P_n are four independent operators commuting with the Dirac operators in the first and second equations (for fixed $J \neq 0$ there are four independent states of two particles with spin 1/2).

Since a_n describe short-range interaction they cannot increase with J. For the majority of mesons we can take a_n as constants independent of J. Only heavy quarkonia demand more complicated a_n, containing decreasing with J contributions.

The choice of P_n is connected with the choice of meson states at fixed J and will be discussed later on.

The solution of the Dirac equations (14) is a 4×4 matrix

$$\Psi = \frac{1}{\sqrt{(1+b_1^2)(1+b_2^2)}} \begin{pmatrix} -b_2\chi\vec{\sigma}\vec{k} & \chi \\ b_1 b_2 \vec{\sigma}\vec{k}\chi\vec{\sigma}\vec{k} & -b_1\vec{\sigma}\vec{k}\chi \end{pmatrix} \tag{16}$$

where

$$b_i = \frac{l_i}{l + \sqrt{m_i l_i / a}}, \quad b_i \to \begin{cases} 1, \ m_i \to 0 \\ 0, m_i \to \infty \end{cases} \tag{17}$$

and χ is an arbitrary normalized 2×2 matrix.

We shall take

$$\chi = \chi_{jMlS}, \tag{18}$$

which are eigenfunctions of $\vec{j}^2, j^{(3)}, \vec{L}^2$ and $\vec{s}^2$ with eigenvalues $j(j+1), M, l(l+1)$ and $S(S+1)$, respectively, where $\vec{j} = \vec{L} + \vec{s}$ and $\vec{s}$ is a 2-dimensional quark spin

$$\vec{s} = \frac{1}{2}\vec{\sigma} \otimes 1 + \frac{1}{2} 1 \otimes \vec{\sigma}^c, \quad \vec{\sigma}^c = \sigma_2\vec{\sigma}\sigma_2 = -\vec{\sigma}^* \tag{19}$$

and 1 stands for the 2×2 unit matrix. These functions can be easily constructed with the help of Clebsh-Gordon coefficients.

The corresponding functions Ψ, denoted by Ψ_{jMlS}, are eigenfunctions of $\vec{J}^2$ and $J^{(3)}$ with eigenvalues $j(j+1)$ and M, respectively, and eigenfunctions of space and charge parities

$$P\Psi_{jMlS} = -(-1)^l \Psi_{jMlS}, \quad \Psi^c_{jMlS} = (-1)^{l+S}\Psi_{jMlS} \tag{20}$$

defined by

$$P\Psi(\vec{k}) = \gamma^0\Psi(-\vec{k})\gamma^0, \quad \Psi^c(\vec{k}) = C\Psi^T(-\vec{k})C^T, \tag{21}$$

where C is the charge conjugation matrix. The Dirac equations (14) are charge conjugation invariant for $m_1 = m_2$.

We shall assume that mesons in the states $\Psi_{jM,j-1,1}$ and $\Psi_{jM,j+1,1}$ do not mix.

This means that mesons with definite C or G-parity are described by the wave functions $\Psi_{jMlS} \equiv \Psi_n$, and the operators P_n in Eq.(15) are projection operators

$$P_n\Psi_m = \delta_{mn}\Psi_n. \tag{22}$$

Eq. (15) for these states takes the form

$$\sqrt{j(j+1)} = K + a_n \tag{23}$$

called the spectral condition. Here K is a known function of m, m_1, m_2 and a, given by Eqs. (10), (11), and (13). As mentioned before a_n can be taken independent of j for all mesons except $c\bar{c}$- and $b\bar{b}$-mesons in the states with $n = jM, j-1, 1$ for which a decreasing with j term must be also taken into account[1].

For strange mesons we consider mixing of the states with $jMj0$ and $jMj1$ for $j > 0$.

The detailed comparison of the spectral condition (15) with experiment[4] and the potential model predictions[5], determination of the model parameters and predictions for j up to 7 are made in Ref.[6]. Here we confine ourself by short comments and illustrative figures.

The light-quark current masses give very small contribution to the spectral condition and can not be determined from this condition. So we use the linear chiral SU_3 model relations $m_u/m_d = 0.55, \;\; m_s/m_d = 20,1$.

The best known meson Regge trajectories are described by $\Psi_{jM,j-1,1}$. The short-range parameters for them depend very weakly on the quark masses. These trajectories allow one to determine the main parameters of the model (masses in MeV)

$$\begin{gathered} a = 0.176 \pm 0.002 \mathrm{GeV}^2 \\ m_s = 224 \pm 7, \;\; m_c = 1440 \pm 10, \;\; m_b = 4715 \pm 20 \\ m_u = 6.2 \pm 0.2, \;\; m_d = 11.1 \pm 0.4. \end{gathered} \tag{24}$$

Fig.1 shows some of these trajectories. For the light-quark mesons the trajectory is practically linear.

For the light-heavy-quark mesons the trajectories are not linear but can be made practically linear by replacing the argument m^2 by $(m-m_h)^2$ where m_h is the heavy quark mass (Fig.2). In the limit $2(m-m_h)/\pi m_h << 1, \quad \pi m_l/2(m-m_h) << 1$, where m_l is the light-quark mass, they must be linear with the slope twice as big as for the light-quark mesons. We see the trajectories are practically linear up to $j=5$ with bigger effective slopes, but the condition for the limit slope is not fulfilled.

For the heavy quarkonia the trajectories are essentially nonlinear, Fig.3.

Nonlinear are also trajectories for Ψ_{jMj0} and $\Psi_{jM,j+1,1}$ states near zero spins, Fig.4. Only one parameter, the short-range contribution a_0 or a_+, is unknown for each of these trajectories. It is determined from the mass of corresponding spin-0 meson.

The constant a_0 for the light-quark mesons is small. According to the linear chiral SU_3 model, it must be proportional to the light-quark masses. We see that the nonlinear trajectory (23) with the universal string tension describes quite well the three mesons π, b_1 and π_2.

The strange mesons $K_0^*(1430), 0^+$ and $K_0^*(1680), 1^-$ are not described by the same wave function $\Psi_{jM,j+1,1}$ (with different j). It seems probable that a new strange 1^- meson exists with mass 1900 MeV which is a partner of $K_0^*(1430)$.

There is not enough data to analyse Ψ_{jMj1} states. So we assume that for nonstrange mesons their short-range parameters a_1 satisfy

$$a_1 = a_0. \tag{25}$$

Experiment confirms this assumption for known mesons composed by light, $s\bar{s}$- and $c\bar{c}$-quarks. If Eq. (25) is fulfilled also for the $b\bar{b}$-mesons then using the known mass of $\chi_{b1}(1P)$ we can estimate the mass of a pseudoscalar $b\bar{b}$-meson $\eta_b(1S), 0^{-+}$ to be 9.50 GeV which is bigger than the mass of $\Upsilon(1S)$.

Linearly extrapolating between $a_0(b\bar{u}) = -0.55$ and $a_0(b\bar{b}) = -0,091$, obtained from Eq. (25), we can find $a_0(b\bar{c}) = -0.41$ and estimate the mass of a pseudoscalar B_c-meson to be 6.40 GeV, which is 0.13 GeV higher than in the potential model[5].

We take into account mixing of Ψ_{jMj0} and Ψ_{jMj1} states only for strange mesons. The trajectories for the mixed states with the mixing angle 36° are shown in Fig.4.

The model allows one to calculate quark velocities and energies and string energy in mesons at rest, Table 1. The light quarks are relativistic and give noticeable contributions to the meson masses. However, the main contribution to the mass "excess" of mesons $D = m - m_1 - m_2$ is given by the gluon string.

Spin structure usually means average values of internal angular momentum variables. The SQM allows one to calculate the spin structure of every meson on leading trajectories, which depends on spin-parity and mass of the meson, string tension and current masses of quarks composing the meson. The spin structure of some mesons is represented in Tables 2 and 3.

Table 1: Energy distribution inside mesons at rest. $v_i(E_i)$ is velocity in c (energy in MeV) of the i-th quark, E_0 is energy of the gluon string in MeV and $D = m - m_1 - m_2$.

Particle, quark content	v_1	v_2	E_1	E_2	E_0	$\frac{E_0}{m}$, %	$\frac{E_0}{D}$, %
$\rho^+, d\bar{u}$	0,98	0,99	53	39	679	88	90
$\pi^+, d\bar{u}$	0,88	0,93	23	16	99	72	82
$B^+, b\bar{u}$	0,07	0,99	4727	46	507	9.6	91
$J/\psi(1S), c\bar{c}$	0,22	0,22	1476	1476	146	4.7	67
$\Upsilon(1S), b\bar{b}$	0,05	0,05	4720	4720	22	0.2	67
$\chi_{b2}(1P), b\bar{b}$	0,18	0,18	4795	4795	324	3.3	67

Table 2: Spin structure of some mesons. Average values of internal angular momentum variables are shown for polarized mesons with $J^{(3)} = M$. [nr]— nonrelativistic limit (quark mass (velocity) is $\infty(0)$), [r] — real case and [ur] — ultrarelativistic limit (quark mass (velocity) is 0(1)).

<table>
<tr><td rowspan="2"></td><td colspan="3">$\overline{S_1^{(3)}}/M$</td><td colspan="3">$\overline{S_2^{(2)}}/M$</td><td colspan="3">$\overline{S^{(3)}}/M$</td><td colspan="3">$\overline{L^{(3)}}/M$</td></tr>
<tr><td>[nr]</td><td>[r]</td><td>[ur]</td><td>[nr]</td><td>[r]</td><td>[ur]</td><td>[nr]</td><td>[r]</td><td>[ur]</td><td>[nr]</td><td>[r]</td><td>[ur]</td></tr>
<tr><td>$\rho^+, u\bar{d}$</td><td rowspan="2">1/2</td><td>0,22</td><td rowspan="2">1/6</td><td rowspan="2">1/2</td><td>0,23</td><td rowspan="2">1/6</td><td rowspan="2">1</td><td>0,45</td><td rowspan="2">1/3</td><td rowspan="2">0</td><td>0,55</td><td rowspan="2">2/3</td></tr>
<tr><td>$K^{*+}, u\bar{s}$</td><td>0,22</td><td>0,42</td><td>0,63</td><td>0,37</td></tr>
</table>

Table 3: Continuation of Table 2. Average values do not depend on the meson polarization.

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<tr><td rowspan="2"></td><td colspan="3">$\overline{\vec{S}^2}$</td><td colspan="3">$\overline{\vec{L}^2}$</td><td colspan="3">$\overline{\vec{S}\vec{L}}$</td></tr>
<tr><td>[nr]</td><td>[r]</td><td>[ur]</td><td>[nr]</td><td>[r]</td><td>[ur]</td><td>[nr]</td><td>[r]</td><td>[ur]</td></tr>
<tr><td>$\pi^+, u\bar{d}$</td><td rowspan="2">0</td><td>0,83</td><td rowspan="2">1</td><td rowspan="2">0</td><td>0,83</td><td rowspan="2">1</td><td rowspan="2">0</td><td>-0,83</td><td rowspan="2">-1</td></tr>
<tr><td>$K^+, u\bar{s}$</td><td>0,79</td><td>0,79</td><td>-0.79</td></tr>
<tr><td>$\rho^+, u\bar{d}$</td><td rowspan="2">2</td><td>1,68</td><td rowspan="2">5/3</td><td rowspan="2">0</td><td>1,87</td><td rowspan="2">7/3</td><td rowspan="2">0</td><td>-0,77</td><td rowspan="2">-1</td></tr>
<tr><td>$K^{*+}, u\bar{s}$</td><td>1,70</td><td>1,17</td><td>-0,43</td></tr>
</table>

We see that the spin structure of these mesons is essentially different from the nonrelativistic case, in particular, the average light-quark spin projections are twice as small.

The spin structure is also different from the ultrarelativistic case when the light-quark current masses are neglected. Unlike the spectral condition, the spin structure is sensitive to the light-quark current masses. Measurement of the spin structure allows one to estimate the light-quark current masses from experiment.

We see also that the flavour SU_3 can be badly broken in the spin structure of spinning mesons. The average value of the $\bar{s}$ spin projection in K^* is 80% bigger than for $\bar{d}$ in ρ.

These results may be relevant to the nucleon spin structure measured in experiment and different from the nonrelativistic quark model predictions.

We see that the gluon string in SQM can account for the quark confinement in mesons.

The string comprises two mechanisms of potential quark models (PQM): confinement potential and constituent quark masses.

Systematic experimental study of meson spectroscopy is important to check the SQM predictions in comparison with the PQM predictions.

Spin structure of light-quark vector mesons in SQM is different from the nonrelativistic quark model (NQM): for the average light-quark spin projections $\bar{S}_{SQM} \cong \frac{1}{2}\bar{S}_{PQM}$.

The flaviour SU_3 can be badly broken in the spin structure in SQM: for s and d quarks $\bar{S}_s \cong 2\bar{S}_d$.

Experimental study of the spin structure may eventually provide experimental estimation of the light-quark current masses.

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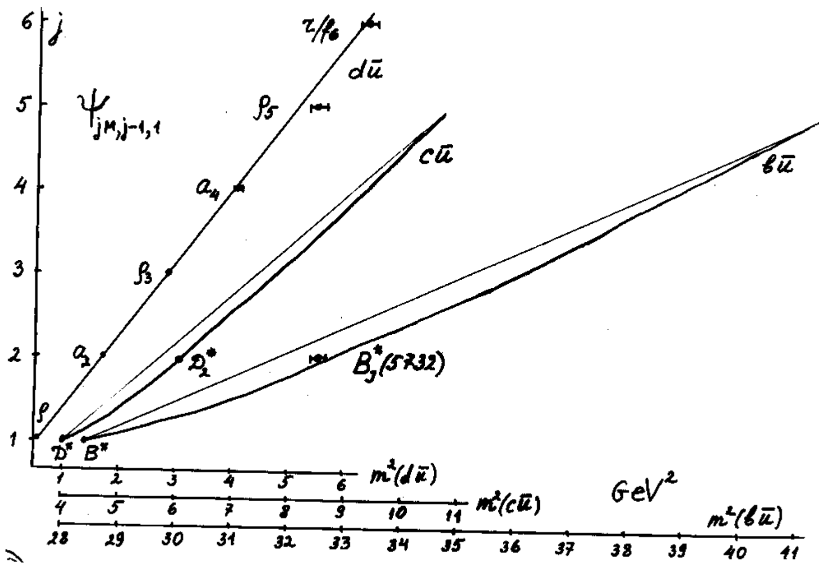

Figure 1: Regge trajectories for mesons with $P = C = (-1)^j$ and $j_{min} = 1$. The argument scales are shifted for each group of mesons. The straight lines for $c\bar{u}$- and $b\bar{u}$-mesons are drawn to show the deviation of the trajectories from linear ones.

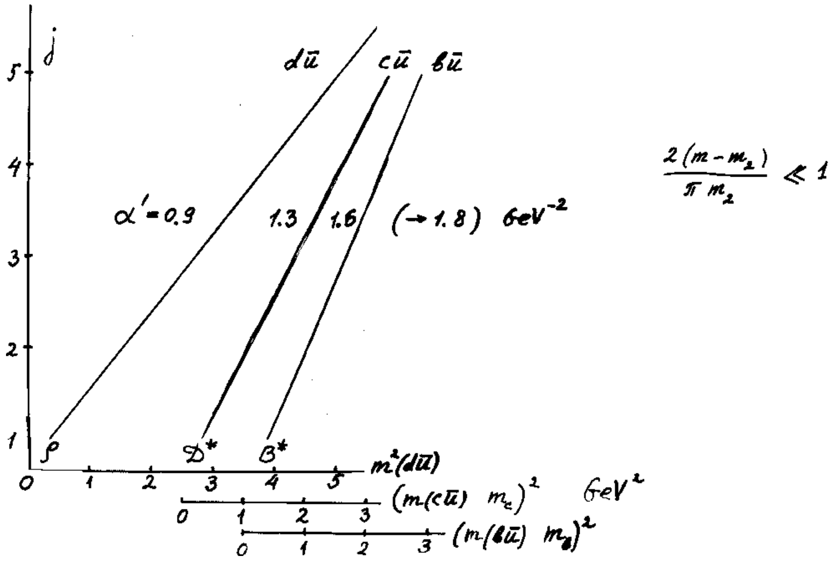

Figure 2: The same as in Fig.1 with changed arguments for $c\bar{u}$- and $b\bar{u}$-mesons.

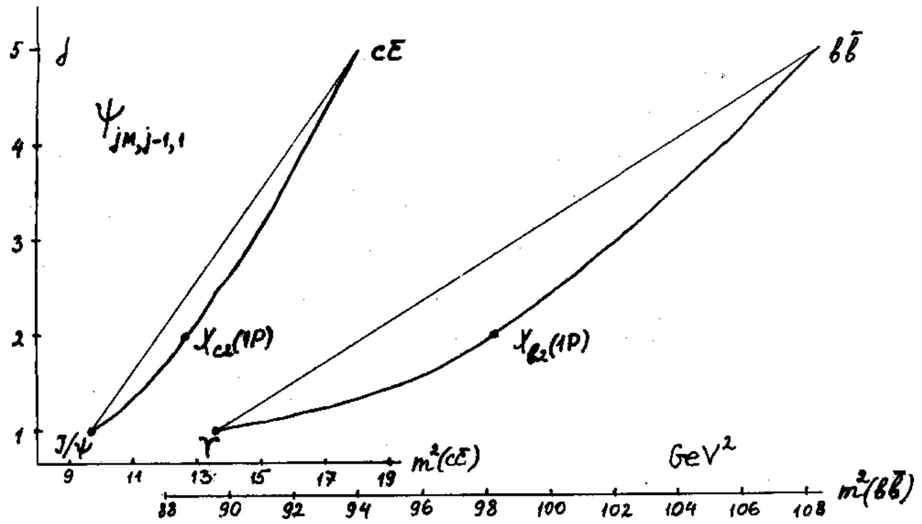

Figure 3: The same as in Fig.1 for heavy quarkonia.

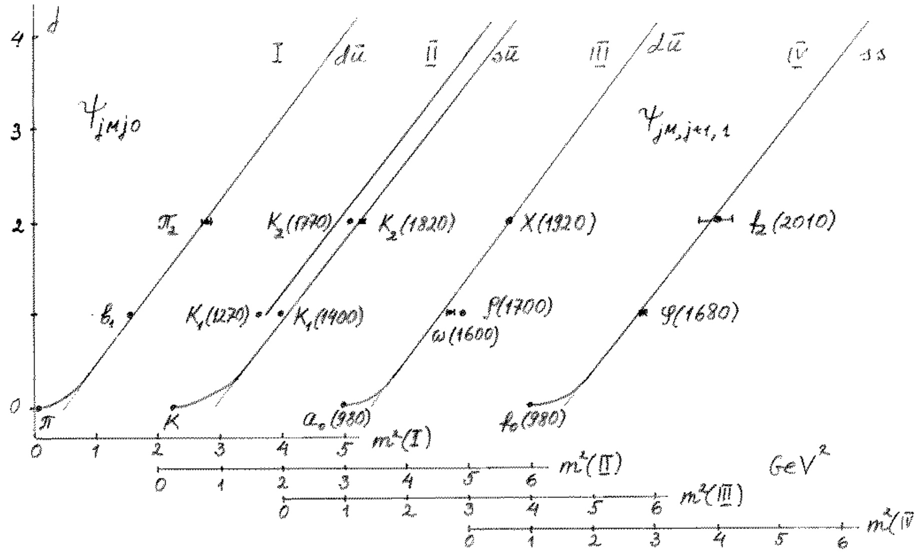

Figure 4: Regge trajectories for light-quark mesons with $-P = C = (-1)^j$ (I), for strange mesons with $P = -(-1)^j$ (II) and for light-quark (III) and $s\bar{s}$-quark (IV) mesons with $P = C = (-1)^j$ and $j_{min} = 0$. The straight lines near $j = 0$ are drawn to show the deviation of the trajectories from linear ones with the universal string tension. The argument scales are shifted for each group of mesons.